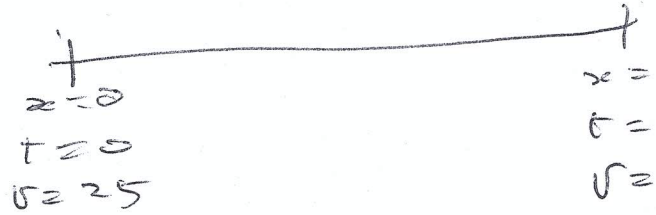
2002
10(b)

(i) $\frac{dv}{dt} = 100 - v$



Link speed and time: $\Rightarrow \frac{dv}{dt} = 100 - v$

$\Rightarrow \int \frac{1}{100-v} = \int dt$ (5)

$\Rightarrow -\ln(100-v)\Big|_{25}^{75} = t\Big|_0^T$

$\Rightarrow -(\ln 25 - \ln 75) = T - 0$ (5)

$\Rightarrow \ln\left(\frac{75}{25}\right) = T$

$\Rightarrow \ln(3) = T$ (5)

(1.1 seconds)

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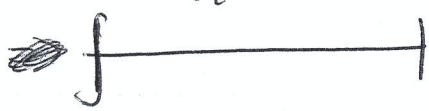
ii) Link distance and time:

$v\frac{dv}{dx} = 100 - v$



$\Rightarrow \int_0^{75} \frac{v}{100-v}\,dv = \int_0^X dx$

$-100\ln(100-v) + 100 - v\Big|_0^{75} = x\Big|_0^X$ (5)

$100\ln(25) + 25 - [-100\ln(100) + 100] = X - 0$

$100\ln 25 + 25 + 100\ln 100 - 100 = X$

$100\ln(4) - 75 = X$

630.3 metres $= X$ (5)

// $\int \frac{v}{100-v}\,dv$

Let $u = 100 - v \Rightarrow v = 100 - u$

$\frac{du}{dv} = -1$

$-du = +dv$

$\int \frac{v}{100-v}\,dv = -\int \frac{100-u}{u}\,du$

$= \int \left(-\frac{100}{u} + \frac{u}{u}\right)du$

$= \int \left(-\frac{100}{u} + 1\right)du$

$= -100\ln u + u$

$= -100\ln(100-v) + 100 - v$